

An experimental Indian gravimetric geoid model using Curtin University's approach

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ABSTRACT

Over the past decade, numerous advantages of a gravimetric geoid model and its possible suitability for the Indian national vertical datum have been discussed and advocated by the Indian scientific community and national geodetic agencies. However, despite several regional efforts, a state-of-the-art gravimetric geoid model for the whole of India remains elusive due to a multitude of reasons. India encompasses one of the most diverse topographies on the planet, which includes the Gangetic plains, the Himalayas, the Thar desert, and a long peninsular coastline, among other topographic features. In the present study, we have developed the first national geoid and quasigeoid models for India using Curtin University's approach. Terrain corrections were found to reach an extreme of 187 mGal, Faye gravity anomalies 617 mGal, and the geoid-quasigeoid separation 4.002 m. We have computed both geoid and quasigeoid models to analyse their representativeness of the Indian normal-orthometric heights from the 119 GNSS-levelling points that are available to us. A geoid model for India has been computed with an overall standard deviation of ± 0.396 m but varying from ± 0.03 to ± 0.158 m in four test regions with GNSS-levelling data. The greatest challenge in developing a precise gravimetric geoid for the whole of India is data availability and its preparation. More densely surveyed precise gravity data and a larger number of GNSS/levelling data are required to further improve the models and their testing.

1. INTRODUCTION

Ideally, Stokes's (1849) integral should be implemented over the entire Earth with continuous gravity anomalies on the geoid and with the condition that there must not be any gravitating masses above it. However, in practice, the availability of gravity observations is limited to a specific area, so the integration domain has to be truncated. Also, the gravity anomalies usually exist discontinuously on or above the Earth's surface so various types of downward continuation and regularisation have been proposed. The gaps between theoretical and practical aspects induce several kinds of errors, which geodesists have tried to reduce, but usually requiring assumptions and approximations.

Based on various ideas, philosophies and numerical

approaches, what we consider the four most commonly used approaches/techniques are adopted for geoid computation experiments in India. (1) Geoid/quasigeoid computation methodology developed at the University of Copenhagen, Denmark (Forsberg 1984, 1985; Forsberg and Tscherning 2008) implemented in the public-domain GRAVSOFT package, (2) the Stokes-Helmert method developed at the University of New Brunswick (UNB), Canada (Vaníček and Kleusberg 1987; Vaníček and Martinec 1994; Vaníček et al. 1999; Ellmann and Vaníček 2007; UNB 2009), (3) the Least Squares Modification of Stokes formula with Additive Corrections (LSMSAC) method developed at the Royal Institute of Technology (KTH), Sweden (Sjöberg 1984, 1991, 2003; Ågren 2004), and (4) geoid/quasigeoid computation methodology developed at Curtin University of Technology (CUT), Australia (Featherstone 2000, 2003; Featherstone et

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al. 1998, 2001, 2011, 2018). There are of course other approaches, such as radial basis functions (e.g., Li 2018; Liu et al. 2020), but perhaps not yet applied as widely. The application areas of the above four approaches are listed in Goyal et al. (2021b).

For India, the first geoid map was developed more than five decades ago. It was based on astrogeodetic observations (Fischer 1961) and with respect to the Everest 1956 ellipsoid (cf. Singh and Srivastava 2018). No more information is available on this geoid, apart from distorted hardcopy contour maps that are difficult to digitise reliably. The levelled height information presently available in India is more than a century old. When these heights were observed, neither the concept of foresight and backsight levelling nor the use of invar staves were considered. Observed gravity values were not available as this was before the development of the low-cost portable relative gravimeter. The Indian vertical datum defined in 1909 was based on constraining the levelling to nine tide-gauges along the Indian coast to zero height (Burrard 1910). We will show later that this approach may have caused a north-south tilt (cf. Fischer 1975, 1977), most probably due to the ocean's time-mean dynamic topography (cf. Featherstone and Filmer 2012).

Frequent seismic activity in various parts of the Indian sub-continent and so-caused crustal movement also necessitate the introduction of a new height system, probably to be based on geopotential numbers and Helmert's orthometric heights (or 'rigorous' orthometric heights as formulated by Santos et al. 2006). The Survey of India (SoI) carried out a re-levelling program (2007 – 2017) with gravity observations at fundamental benchmarks to provide a densified network of Helmert's orthometric heights as a part of the Redefined Indian Vertical Datum 2009 (G&RB 2018; Singh 2018). However, these data are not yet in the public domain, so we are unable to use them to validate our geoid and quasigeoid models. In addition, the national geodetic agencies have proposed to compute a precise national geoid model to serve as the new vertical datum for the country. This can be viewed as following the suit of New Zealand (LINZ 2016), Canada (Véronneau and Huang 2016), and the USA (NGS 2017, 2019). Such an approach is being considered in many other countries too.

Researchers and government organisations have made some efforts to develop local gravimetric geoid models for regions in India (Singh 2007; Carrion et al. 2009; Srinivas et al. 2012; Mishra and Ghosh 2016; Singh and Srivastava 2018), but only using the GRAVSOFTE package with residual terrain modelling (Forsberg 1985). Despite these efforts, a state-of-the-art national gravimetric geoid model for the whole India remains elusive (Goyal et al. 2017). Therefore, in this study, we present the first-ever nationwide geoid and quasigeoid computation results over India with the available data sets using the CUT method implemented using our own computation package developed in MATLAB™.

2. DATASETS

2.1 Terrestrial Gravity

Pointwise observed gravity data is confidential in India. Therefore, with this predicament, we obtained a grid of Indian terrestrial gravity data from GETECH (<https://getech.com/>) that is claimed to come from the Gravity Map Series of India (GMSI), a joint project of five Indian organisations, viz., SoI, Geological Survey of India (GSI), Oil and Natural Gas Corporation (ONGC), National Geophysical Research Institute (NGRI), and Oil India Limited (cf. Tiwari et al. 2014). The GETECH gravity data comprises a $0.02^\circ \times 0.02^\circ$ grid of simple Bouguer gravity anomalies over India (except a few regions in northern India), with an overall estimated precision of ± 1.5 mGal (GETECH 2006). According to the GETECH manual for Indian gravity data, they used (1) the normal gravity formula from WGS84 (NIMA 2000)

$$\gamma_{0_WGS84} = 978032.67714 \left(\frac{1 + 0.00193185438639 \sin^2 \phi}{\sqrt{1 - 0.00669437999013 \sin^2 \phi}} \right) \text{mGal} \quad (1)$$

(2) a second-order free-air correction given by

$$\delta g_{FAC}^{GETECH} = (0.3083293357 + 0.0004397732 \cos^2 \phi) h - 7.2125 \times 10^{-8} h^2 \text{ mGal} \quad (2)$$

(3) the following atmospheric correction (Ecker and Mittermayer 1969)

$$\delta g_{atm}^{GETECH} = \begin{cases} 0.87 e^{-0.116H^{1.047}} \text{ mGal}, & H > 0 \text{ km} \\ 0.87 \text{ mGal}, & H \leq 0 \text{ km} \end{cases} \quad (3)$$

and (4) the simple planar Bouguer correction

$$\delta g_{BC}^{GETECH} = -0.04191 \rho H \text{ mGal} \approx -0.1119H \text{ mGal} \quad (4)$$

where γ_{0_WGS84} is normal gravity on the WGS84 level ellipsoid, δg_{FAC}^{GETECH} is the free-air correction, ϕ is the geodetic latitude, h is the ellipsoidal height (in m), H is the elevation [in km for Eq. (3) and m for Eq. (4)], δg_{atm}^{GETECH} is the atmospheric correction, δg_{BC}^{GETECH} is the planar Bouguer correction and ρ is the constant topographical density of 2670 kg m^{-3} . We re-computed the free-air gravity anomalies (Δg) from the GETECH data so as to be more compatible with the CUT approach by using

$$\Delta g = \Delta g_{SBA}^{GETECH} + 0.1119H + \gamma_{0_WGS84} - \delta g_{FAC}^{GETECH} - \delta g_{atm}^{GETECH} - \gamma_{0_GRS80} + \delta g_{FAC}^{CUT} + \delta g_{atm}^{CUT} \quad (5)$$

where, Δg_{SBA}^{GETECH} are simple Bouguer anomalies from GETECH and

$$\delta g_{FAC}^{CUT} = \gamma_{0_GRS80} \left[\frac{2}{a} (1 + f + m - 2f \sin^2 \phi) H - \frac{3}{a^2} H^2 \right] \quad (6)$$

$$\gamma_{0_GRS80} = \gamma_a \left(\frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \right) \quad (7)$$

$$\delta g_{atm}^{CUT} = 0.871 - 1.0298 \times 10^{-4} H + 5.3105 \times 10^{-9} H^2 - 2.1642 \times 10^{-13} H^3 + 9.5246 \times 10^{-18} H^4 - 2.2411 \times 10^{-22} H^5 \quad (8)$$

For GRS80, $a = 6378137$ m, $e^2 = 0.0066943800229$, $m = 0.0034478600308$, $f = 1/298.257222101$ and $\gamma_a = 978032.67715$ mGal, $k = 0.001931851353$ (Moritz 2000). The descriptive statistics of the differences between the free-air anomalies from the GETECH data and re-computed free-air anomalies are (in mGal): min = -0.001, max = 0.188, mean = 0.002, STD = ± 0.007 . It should be noted that we have used H instead of h (ellipsoidal heights) in Eq. (2) because we believe that there might be a typographical error in the GETECH manual. The rationale being that with the

use of h we would obtain gravity disturbances and not gravity anomalies (cf. Hackney and Featherstone 2003). A blanket accuracy estimate of the reconstructed free-air anomalies from the GETECH Bouguer anomalies is ± 2.4 mGal, calculated using the DEM error in the CUT reconstruction technique as per $\sigma_{FA} = \sqrt{(1.5 \times 10^{-5})^2 + (2\pi G\rho \times 17.3)^2}$.

For the oceanic regions surrounding India, we used free-air gravity anomalies (Version 28.1) from the Scripps Institute of Oceanography (SCRIPPS, https://topex.ucsd.edu/marine_grav/mar_grav.html) which has an overall root mean square error of ± 1.23 mGal (Sandwell et al. 2021). The SCRIPPS data is also accompanied with an error grid that we have shown, for our study area, in Fig. 1. The data contains a $1' \times 1'$ grid that also covers the land, but we used the SCRIPPS data only for the oceanic region because the land data, in the SCRIPPS dataset, is from EGM2008 to avoid aliasing (Gibbs fringing) at the coasts.

We do not have gravity data from the countries neighbouring India and a well distributed sufficient data coverage is not available in the Bureau Gravimetric International (<https://bgi.obs-mip.fr/>) archives either (Country: no. of gravity data points - Pakistan: 1270, Bangladesh: 25, Sri Lanka: 48, Myanmar: 71, Afghanistan: 1649, China: 446, Nepal: 617, and Bhutan: 0.). Therefore, we constructed a $0.02^\circ \times 0.02^\circ$ grid of free air anomalies over land using EGM2008 (Pavlis et al. 2012, 2013) up to degree and order (d/o) 900 to fill-in the land gravity anomaly data in and around India where the GETECH data is not available, including Nepal, China, Pakistan, Sri Lanka, Bangladesh, Bhutan, Afghanistan, and Myanmar. The specific d/o 900

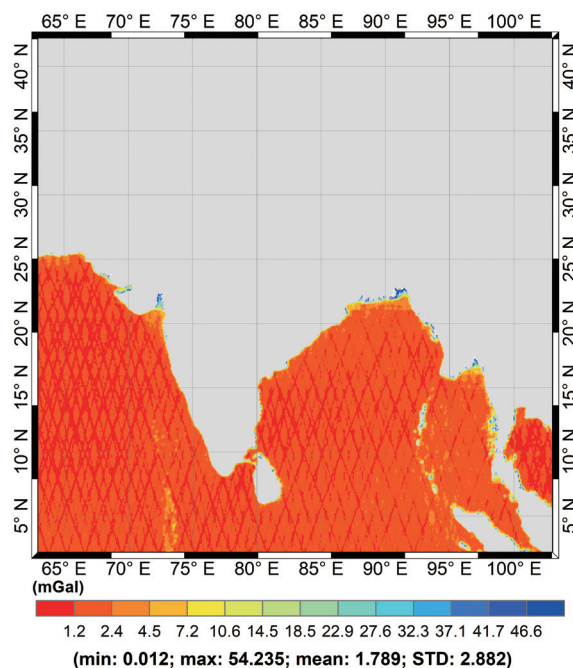


Fig. 1. Error map of the SCRIPPSv28.1 marine gravity-anomaly data (units in mGal).

was chosen because EGM2008 uses proprietary data up to d/o 900 (Pavlis et al. 2013).

As discussed next, we merged these three datasets to get a complete free-air gravity anomaly grid of $0.02^\circ \times 0.02^\circ$, avoiding aliasing or the contamination of land data (both GETECH and EGM2008 individually) with the marine data or vice-versa.

There exist numerous sophisticated space-domain and frequency-domain methods for merging heterogeneous gravity anomaly datasets (e.g., Strykowski and Forsberg 1998; Olesen et al. 2002; Catalao 2006; McCubbine et al. 2017). However, we chose to work with the comparatively straightforward CUT space-domain method (cf. Featherstone et al. 2011, 2018). This choice is somewhat arbitrary because we are working with the land gravity of unknown quality, and the strategy that we use has already been implemented in the computation of the Australian quasigeoid, which is an island nation and approximately 2.3 times larger than India. Other methods can also be tested, but it is left for the time when sufficient marine and airborne gravity data along with reliable terrestrial gravity data will be available over India.

In the adopted method, the GETECH-derived free-air anomaly grid is superimposed over the EGM2008 (d/o 900) derived gravity anomalies. The gravity anomalies of the latter dataset at the overlapping grid nodes are replaced by the gravity anomalies from the former dataset. As a result, a $0.02^\circ \times 0.02^\circ$ grid of gravity anomalies on the land is obtained.

To concatenate the land and marine gravity anomaly data, $1' \times 1'$ gravity anomalies in the ocean are clipped (or separated) from the complete SCRIPPS dataset, i.e., on both ocean and land. It is then block averaged to the $0.02^\circ \times 0.02^\circ$ grid and is superimposed with the land gravity anomaly grid. The former values were replaced by the latter at overlapping nodes to obtain the $0.02^\circ \times 0.02^\circ$ grid of the merged gravity anomalies. Figure 2 shows the merged free-air gravity anomaly map. To check for any discontinuities at the edges of the merged datasets, we computed and plotted the arctangent (Fig. 3a) and logarithmic (Fig. 3b) values of the gradients of the merged data. We observe no clear visual indication of any discontinuities at the boundaries of the merged data, but also partially due to the ruggedness of the dataset in our study area that can be obscuring.

2.2 Digital Elevation Model

The Digital Elevation Model (DEM) is another important input in geoid computation. It is mainly used to compute the topographical effects (e.g., Forsberg 1984). Thus, a precise high-resolution DEM should be used. We would like to mention here that DEM is generally used synonymously with a Digital Surface Model (DSM) (e.g., SRTM, ASTER), but this should be avoided. Quantification of the differences in the topographical effect with the use of DEM

and DSM has been investigated by Yang et al. (2019). Since India does not have a national DEM, therefore, after a DEM/DSM analysis (Goyal et al. 2021a), it was decided to work with the best available DEM over India, i.e., the MERIT 3" \times 3" DEM (Yamazaki et al. 2017), for our computations. Though the accuracy of the MERIT DEM varies considerably (± 11.7 to ± 47.3 m) over different landforms in India, an overall estimate for the whole of India is ± 17.3 m (Goyal et al. 2021a).

2.3 GNSS-Levelling

India has different horizontal and vertical control networks. Therefore, presently there are only a limited number of ground control points where we have the geodetic coordinates (latitude, longitude, ellipsoidal heights) and levelled heights. Moreover, due to several restrictions on the datasets, only a few of these available data points were available to us (Fig. 4). The datasets in the Uttar Pradesh west (UP west) and Uttar Pradesh east (UP east) regions were procured from SoI, while the datasets over Hyderabad and Bangalore have been retrieved from Mishra (2018), who also used the SoI dataset. According to Mishra (2018), horizontal and vertical precisions of GNSS data are within ± 12 to ± 26 mm and ± 31 to ± 53 mm, respectively. The vertical precision of the levelling heights is not known to us, but they are from the high precision first level net of India. These heights are from the Indian Vertical Datum 1909 (Burrard 1910) and are based on the normal-orthometric height system, while those on Indian Vertical Datum 2009 (G&RB 2018) are based on Helmert's orthometric height system. We have not been provided with a clear indication on which heights have been provided to us, and therefore, due to this anonymity of the height system, we consider the levelling heights to be in the normal-orthometric height system (Jekeli 2000; Featherstone and Kuhn 2006).

3. METHODOLOGY AND RESULTS

An overview of the CUT methodology for computing the geoid undulations is shown by a flowchart in Fig. 5. The CUT method primarily computes the quasigeoid using the analytical continuation solution (Moritz 1971, 1980) of Molodensky's problem (Molodensky et al. 1962). Moritz (1971) showed that Molodensky's G_1 term can be approximated by the planar terrain correction (TC), which also needs an additional term that is equal to the first-order indirect effect (FOIE). We could not adopt the full CUT method-based reconstruction of Faye anomalies (Featherstone and Kirby 2000) because we already have gridded data, whereas CUT grids point Bouguer anomalies. Instead, we added the block averaged $0.02^\circ \times 0.02^\circ$ grid of TCs (Fig. 6a) to the free-air gravity anomaly grid to calculate area-mean Faye anomalies (Fig. 6b). The block-averaged $0.02^\circ \times 0.02^\circ$

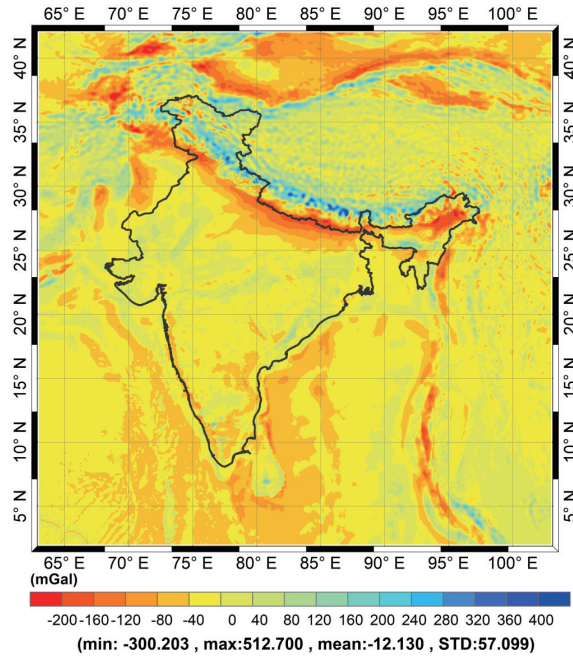


Fig. 2. Merged gravity anomaly data from GETECH, EGM2008 (d/o 900), and SCRIPPS (units in mGal).

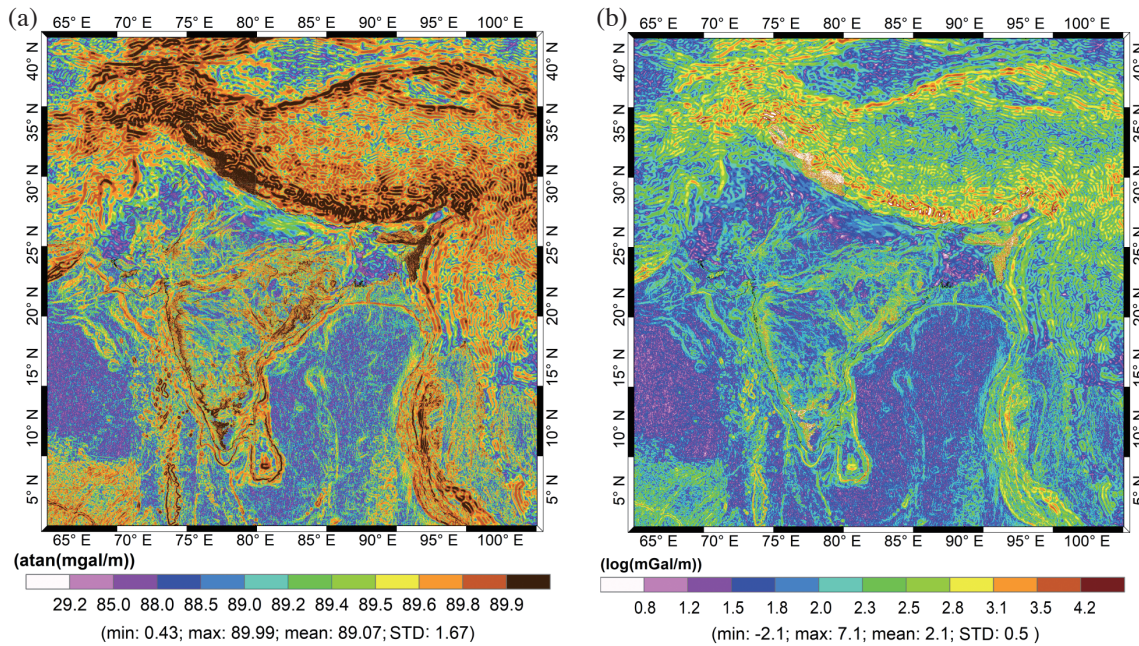


Fig. 3. Arctangent (a) and logarithmic (b) plot of gradients of merged gravity anomaly data to attempt to identify discontinuities at the edges of the merged grids.

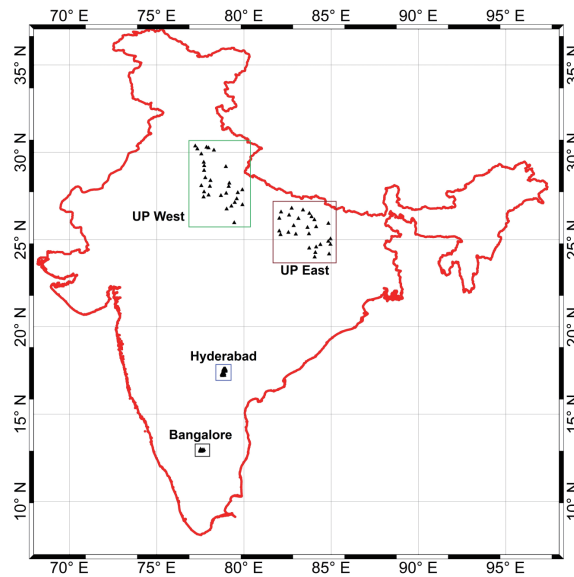


Fig. 4. Spatial coverage of the available 119 GNSS/levelling data points.

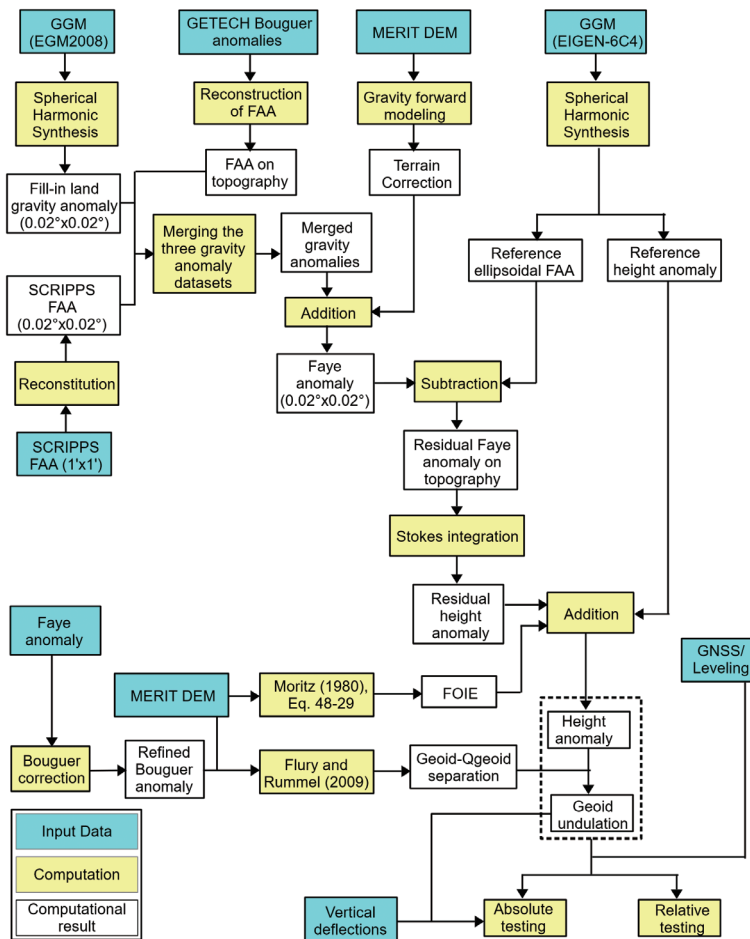


Fig. 5. Flowchart of the CUT methodology of geoid/quasigeoid computation as applied in India for these experiments.

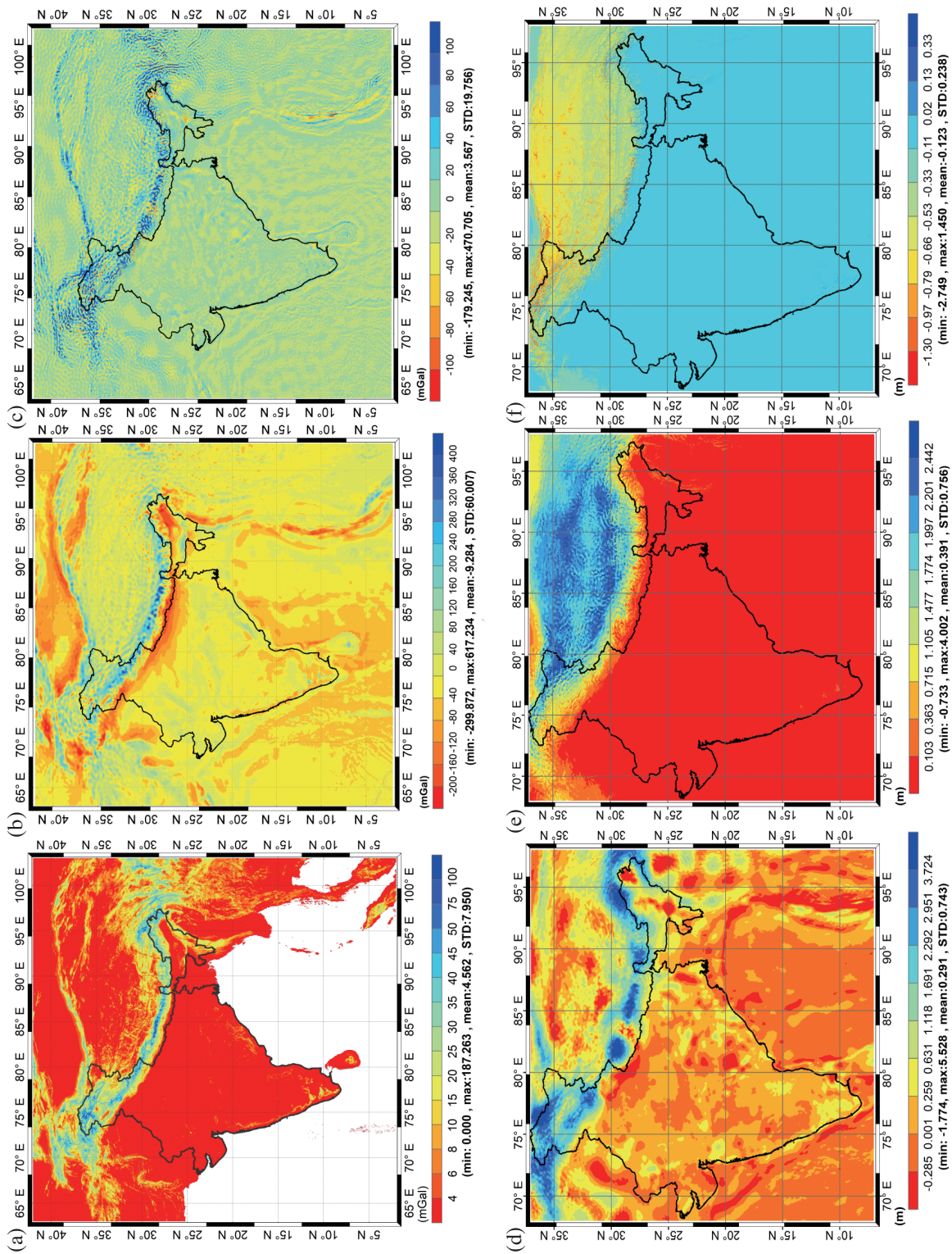


Fig. 6. (a) Block averaged planar TC, (b) Faye anomaly, (c) residual Faye anomaly, (d) residual quasigeoid ($M = 80$ and $\psi = 1.5^\circ$), (e) quasigeoid-geoid separation term (Flury and Rummel 2009), (f) difference in quasigeoid-geoid separation term (Heiskanen and Moritz 1967; Flury and Rummel 2009) (all on a $0.02^\circ \times 0.02^\circ$ grid) [units of (a), (b), (c) in mGal and (d), (e), (f) in m].

TC grid was constructed from the 3" × 3" TC grid computed with the MERIT DEM using the Optimal Separating Radius (OSR) in the spatial-spectral combined method suggested by Goyal et al. (2020). This method of TC guarantees the full convergence of the TC solution, i.e., down to < 1 μGal.

A different approach is used in the CUT method to apply ellipsoidal correction. Unlike other geoid computation strategies considered (UNB or KTH; cf. Huang et al. 2003; Ellmann 2005), the CUT method computes ellipsoidal area-mean free-air gravity anomalies on the topography using a Global Geopotential Model (GGM) (Featherstone et al. 2018). These are subtracted from the mean Faye gravity anomalies to obtain residual gravity anomalies (Fig. 6c), which are then Stokes-integrated with the Featherstone-Evans-Olliver (FEO) modified kernel (Featherstone et al. 1998) to obtain the residual height anomalies. The FEO kernel, a deterministic modifier, is the combination of the Meissl (1971) and Vaníček and Kleusberg (1987) modifiers that simultaneously reduces the truncation error and improves the rate of convergence to zero of the series expansion of the truncation error (cf. Featherstone et al. 1998; Featherstone 2003). Additionally, the spherical reference radius in the Stokes integration is set equal to the geocentric ellipsoidal radius of the computation point, and this negates the need for further ellipsoidal corrections to Stokes's integral (Claessens 2006).

The residual height anomalies were computed using the following parameter-sweeps of the modification degree (M): 0, 40, 80, 120, 160, 200, 240, 280, 300, and integration cap radius (ψ): 0.2°, 0.5°, 0.75°, 1°, 1.5°, 2° (e.g., Fig. 6d for $M = 80$, $\psi = 1.5^\circ$). The reference height anomalies on the topography are computed using GGMs with a zero-degree term (N_0) from the generalised Bruns's formula [Eq. (9)] (Heiskanen and Moritz 1967) calculated for each latitude parallel, which are added to the residual height anomalies to obtain the required height anomalies. An inconsistent use of Eq. (9) can cause an error of ~1 m in the computed geoid undulations/height anomalies. We used normal potential U_0 (= 62636860.85 m² s⁻²) from GRS80 (Moritz 2000) and the geopotential W_0 (= 62636853.4 m² s⁻²) from IHRS (Sánchez et al. 2016).

$$N_0 = \frac{GM_G - GM_E}{r\gamma_0} - \frac{W_0 - U_0}{\gamma_0} \quad (9)$$

As a small modification to the original CUT method, we added the $FOIE = \pi G\rho H^2/\gamma$ [Moritz 1980, Eqs. (48) – (29); Heiskanen and Moritz 1967, chapter 8] to the computed height anomalies. We note that the negative sign is sometimes omitted (e.g., Sjöberg 2000; Hwang et al. 2020).

The geoid undulations are calculated by adding the quasigeoid-geoid separation term (Fig. 6e; Flury and Rummel 2009) to the height anomalies. The more rigorous quasi-

geoid-geoid separation term from Flury and Rummel (2009) differs quite considerably from the approximate formula given in Heiskanen and Moritz (1967, p 328). The difference in the quasigeoid-geoid separation term from the two methods is shown in Fig. 6f. Acknowledging that the number and distribution of the GNSS/levelling data points are not sufficient for reliable fitting (Kotsakis and Sideris 1999; Fotopoulos 2003), we have not presented hybrid geoid and hybrid quasigeoid models for this experiment over India.

The geoid should be validated with orthometric [Helmert or rigorous (Santos et al. 2006)] heights and the quasigeoid validated with normal heights. A more rigorous validation approach would be to convert the normal-orthometric heights to Helmert's orthometric height and normal heights for validating geoid and quasigeoid, respectively. Examples of this are Foroughi et al. (2017) and Janák et al. (2017) over Auvergne, France, where normal heights were converted to rigorous heights for validation of their developed geoid models. However, Indian levelled heights are based on the normal-orthometric height system for which there is no specific choice of reference surface, i.e., either geoid or quasigeoid. Therefore, we are only able to "validate" the developed geoid and quasigeoid models with the Indian normal-orthometric heights on an uncertain vertical datum (section 2.3).

Absolute and relative testing (Featherstone 2001) of both height anomalies and geoid undulations are done in this study. The absolute testing is realised through point-wise subtraction of gravimetric geoid undulations obtained using Stokesian integration (N) and the geometrical geoid undulation ($h - H$) obtained using GNSS/levelling data [Eq. (10)].

$$\mathcal{E}_i^{abs} = N_i - (h_i - H_i) \quad \forall i = 1, 2, 3, \dots, n \quad (10)$$

where n is the total number of discrete GNSS/levelling data points. It is important to acknowledge that absolute accuracy is only an assumption. This is principally because the levelled heights that refer to the local vertical datum are not necessarily coincident with the geoid. This has been discussed in detail by Featherstone (2001). The descriptive statistics of \mathcal{E}_i^{abs} are in Table 1.

The relative testing of geoid and quasigeoid [Eq. (11)] is an analysis tool to investigate their gradients. This type of analysis is of more interest to land surveyors who use relative GNSS baselines and a geoid/quasigeoid gradients as a replacement for the more time-consuming differential levelling.

$$\mathcal{E}_{ij}^{rel} = \Delta N_{ij} - (\Delta h_{ij} - \Delta H_{ij}) \quad \forall i, j = 1, 2, 3, \dots, n; i \neq j \quad (11)$$

The descriptive statistics of \mathcal{E}_{ij}^{rel} , and the ratio of mean differences to the mean baseline length in parts per million

Table 1. Statistics for the region-wise geoid/quasigeoid (for $M = 80$ and $\psi = 1.5^\circ$) absolute testing (units in m).

Region (no. of points)	Geoid				Quasigeoid			
	min	max	mean	STD	Min	Max	Mean	STD
India (119)	-0.897	0.788	-0.171	± 0.396	-0.906	0.726	-0.185	± 0.389
UP west (29)	-0.897	-0.154	-0.532	± 0.138	-0.906	-0.164	-0.548	± 0.142
UP east (27)	-0.712	-0.338	-0.521	± 0.114	-0.711	-0.340	-0.523	± 0.114
Hyderabad (56)	-0.385	0.501	0.070	± 0.158	-0.400	0.488	0.057	± 0.158
Bangalore (7)	0.709	0.788	0.751	± 0.030	0.645	0.726	0.690	± 0.032

(average ppm in mm km^{-1}) for the geoid and quasigeoid are in Table 2.

The variation of standard deviation in the Indian geoid and quasigeoid models, on testing with GNSS/levelling data, for different combinations of modification degree and integration cap are shown in Figs. 7a and b, respectively. Table 1 depicts the region-wise (UP west, UP east, Hyderabad, Bangalore, and all together) descriptive statistics for the geoid and quasigeoid for the combination of $M = 80$ and $\psi = 1.5^\circ$. Though the standard deviation for the whole of India is smaller with the combination of $M = 40$ and $\psi = 1.5^\circ$ compared to $M = 80$ and $\psi = 1.5^\circ$ (cf. Fig. 7), standard deviations for the four individual regions are less than or equal to the combination of $M = 80$ and $\psi = 1.5^\circ$ compared to $M = 40$ and $\psi = 1.5^\circ$. Therefore, $M = 80$ and $\psi = 1.5^\circ$ was chosen to present our results. The results of the relative testing are shown in Figs. 8a and b, and Table 2. The computed Indian gravimetric geoid (IndGG-CUT2021) and corresponding contours (at a 2-m contour interval) are shown in Figs. 9a and b, respectively.

4. DISCUSSION, RECOMMENDATIONS AND CONCLUSIONS

Though the number (119) and the distribution (Fig. 4) of the GNSS/levelling data points are insufficient to draw concrete conclusions about the quality of the computed gravimetric geoid and quasigeoid models, the following are some major observations from our experimental results:

- (1) Since the study area comprises the most complex topography varying from the Himalayas to the Gangetic plains and a long peninsular coastline, Fig. 6 possibly depicts the extreme (maximum and minimum) values of planar TC, Faye gravity anomaly, and quasigeoid-geoid separation on the planet.
- (2) From the viewpoint of the “cm-level accurate” geoid, Fig. 6f suggests that a more rigorous method (e.g., Flury and Rummel 2009) should be preferred for calculating the quasigeoid-geoid separation over a simple approximate formula (e.g., Heiskanen and Moritz 1967). There exist other formulas for the quasigeoid-geoid separation

term (e.g., Sjöberg 2010; Foroughi and Tenzer 2017), but they are not tested here.

- (3) Figure 7 suggests that the FEO kernel (Featherstone et al. 1998) is not numerically unstable for higher modification degrees, as shown in Featherstone (2003), Li and Wang (2011), Featherstone et al. (2018), and Claessens and Filmer (2020). However, this observation can also result from our choice of parameter sweeps and limited datasets for validation, thus requiring further investigation.
- (4) Generally, standard deviations versus GNSS/levelling are large for lower modification degrees and larger integration radii (Featherstone et al. 2018; Claessens and Filmer 2020). However, Fig. 7 shows an opposite trend in India, with smaller standard deviations for lower modification degrees and larger integration radii. This is primarily attributable to the north-south tilt in the India height datum (cf. Table 1). However, the smaller number of GNSS/levelling data and their poor distribution are also likely to contribute to this observation.
- (5) Figure 7 shows that the Indian levelling heights are marginally better referred to the quasigeoid (std = ± 0.389 m) than the geoid (std = ± 0.396 m). However, Table 1 shows that the geoid has an equal or better precision estimate than the quasigeoid (in terms of standard deviation) in each of the four regions individually. The difference in the standard deviations of the quasigeoid and geoid comparison for the whole of India seems to be a consequence, mostly, of the smaller mean of the quasigeoid (0.690 m) than the geoid (0.751 m) comparison over Bangalore. Also, with the given precision estimate of the data points, there can yet be no preferred choice between geoid or quasigeoid for the Indian vertical datum. Hence, a larger set of data points are needed for any possible claim of reference surface for India. Though the overall standard deviation of the computed geoid/quasigeoid (Table 1) is $\sim \pm 0.40$ m, it varies from $\sim \pm 0.03$ to $\sim \pm 0.16$ m if only evaluated individually in the four small test regions.
- (6) Table 2 indicates that the largest misclosures in Fig. 8 are probably due to the tilt in the Indian height datum and the relative closeness of data points in Hyderabad and Bangalore, which also explains the larger ppm

- values found in those regions. Spikes in Figs. 8a and b at distances of approximately (0 – 50), (450 – 550), (900 – 1200), and (1200 – 1900) km are due to the errors and differences (north-south tilt) in the baselines for (Bangalore and Hyderabad, individually), (Bangalore to Hyderabad), (UP west, UP east to Hyderabad), and (UP west, UP east to Bangalore), respectively.
- (7) On comparison of validations of the Indian gravimetric geoid with the CUT method and the GGM (Table 3), it is observed that though the overall mean values are improved for all regions except Bangalore, an improvement in the standard deviation beyond ± 0.01 m is observed only for UP east. However, the standard deviation of gravimetric geoid in UP west is degraded by ± 0.03 m as compared to the EIGEN-6C4. A degradation in the standard deviation of the gravimetric geoid is also observed in Featherstone and Sideris (1998). This was, and similarly is, attributed to errors in either one or more of terrestrial gravity data, GGMs and the GNSS/levelling data. There is little to no improvement with the inclusion of the terrestrial gravity data with the CUT method because it makes use of the highest available degree-order GGMs. Also, the GETECH data is possibly already included in the high degree-order GGM (e.g., EGM2008, Pavlis et al. 2012, 2013).
- (8) The Faye gravity anomaly (Fig. 6b), geoid (Fig. 9a), and

contour map (Fig. 9b) somewhat depict the separation line of the Indian and the Eurasian plate. Thus, the results presented in this study could be important for geophysical studies. The contour pattern around the location of 24°N and 82°E seems intriguing for some gravimetric studies in that region. It should also be noted that the area comprises one of the largest coalfields of India with the thickest and different varieties of coal seams.

As a final remark, first experimental geoid and quasigeoid models for India have been computed with a standard deviation of ± 0.396 and ± 0.389 m, respectively, with respect to a small number of test regions. However, for the four regions individually, the standard deviation varies from ± 0.030 to ± 0.158 m for the geoid and ± 0.032 to ± 0.158 m for the quasigeoid. Though all the results presented herein are the first from India, the geoid/quasigeoid must be improved with dense, precise gravity data. Moreover, a larger number of GNSS/levelling data points must become available for more rigorous validation of the gravimetric geoid/quasigeoid. For the re-computation of the Indian geoid/quasigeoid with the CUT method and additional gravity data, the TC and the quasigeoid-geoid separation term need not be computed again unless a high-resolution and more precise DEM is available. Further, due to the complexities of the Indian topography and geomorphic characteristics, other geoid/quasigeoid computation strategies should also be tested over India.

Table 2. Statistics for the region-wise geoid/quasigeoid (for $M = 80$ and $\psi = 1.5^\circ$) relative testing.

Region (Mean distance)	Geoid					Quasigeoid				
	min (m)	max (m)	mean (m)	STD (m)	Average ppm	min (m)	max (m)	mean (m)	STD (m)	Average ppm
India (713.46 km)	-0.620	1.684	0.373	± 0.418	3.371	-0.625	1.632	0.368	± 0.408	3.362
UP west (197.28 km)	-0.605	0.743	0.040	± 0.191	1.111	-0.602	0.742	0.057	± 0.193	1.118
UP east (169.33 km)	-0.374	0.367	0.015	± 0.161	1.048	-0.372	0.367	0.018	± 0.161	1.052
Hyderabad (18.67 km)	-0.620	0.886	-0.031	± 0.221	13.032	-0.625	0.888	-0.032	± 0.221	13.025
Bangalore (14.08 km)	-0.074	0.079	-0.005	± 0.044	3.113	-0.077	0.081	-0.008	± 0.046	3.281

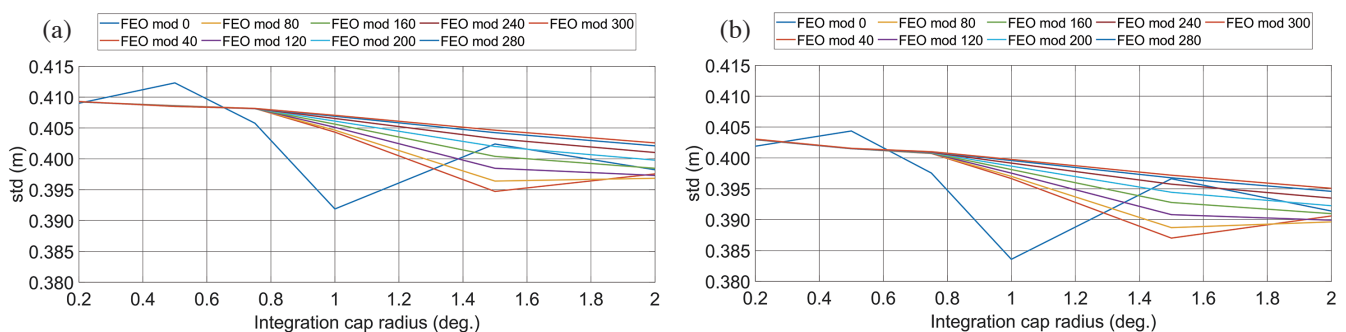


Fig. 7. Standard deviation of (a) geoid and (b) quasigeoid of India for different combinations of modification degree and integration cap (units in m).

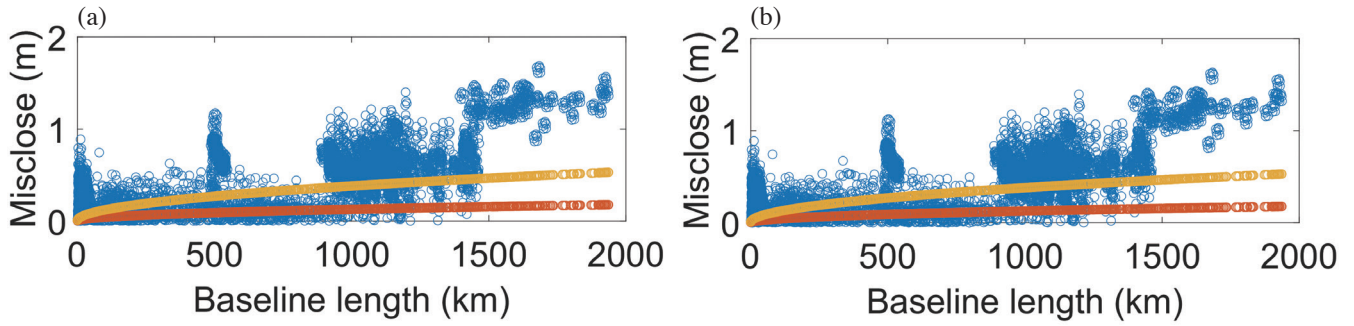


Fig. 8. Magnitude of relative differences (blue circles) for the (a) geoid and (b) quasigeoid. Orange and yellow circles represent the maximum permissible in-field misclose for Indian high-precision ($k = 3$) and double tertiary ($k = 12$) levelling for each baseline, respectively (units in m).

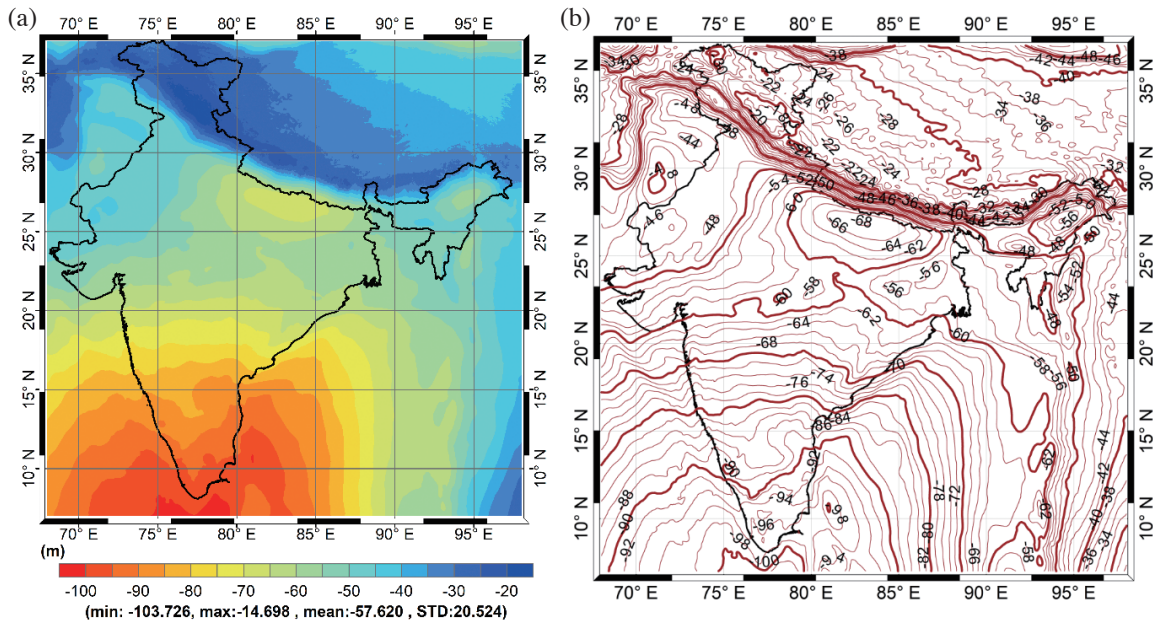


Fig. 9. (a) Indian gravimetric geoid computed using the CUT method (units in m), and (b) corresponding 2 m geoid contours.

Table 3. Comparison of EIGEN-6C4 and IndGG-CUT2021 validated with GNSS/levelling data (units in m).

		min	max	mean	STD
India	EIGEN-6C4	-1.203	0.463	-0.428	± 0.410
	IndGG-CUT2021	-0.897	0.788	-0.171	± 0.396
UP west	EIGEN-6C4	-1.203	-0.643	-0.870	± 0.105
	IndGG-CUT2021	-0.897	-0.154	-0.532	± 0.138
UP east	EIGEN-6C4	-1.034	-0.361	-0.742	± 0.144
	IndGG-CUT2021	-0.712	-0.338	-0.521	± 0.114
Hyderabad	EIGEN-6C4	-0.612	0.258	-0.154	± 0.157
	IndGG-CUT2021	-0.385	0.501	0.070	± 0.158
Bangalore	EIGEN-6C4	0.379	0.463	0.422	± 0.029
	IndGG-CUT2021	0.709	0.788	0.751	± 0.030

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